6.1 Slope Fields and Euler's Method

- **Use initial conditions to find particular solutions of differential equations.**
- **Use slope fields to approximate solutions of differential equations.**
- **Use Euler's Method to approximate solutions of differential equations.**

General and Particular Solutions

In this text, you will learn that physical phenomena can be described by differential equations. Recall that a **differential equation** in x and y is an equation that involves x, y, and derivatives of y. For example,

$$2xy' - 3y = 0 \qquad \text{Differential equation}$$

is a differential equation. In Section 6.2, you will see that problems involving radioactive decay, population growth, and Newton's Law of Cooling can be formulated in terms of differential equations.

A function $y = f(x)$ is called a **solution** of a differential equation if the equation is satisfied when y and its derivatives are replaced by $f(x)$ and its derivatives. For example, differentiation and substitution would show that $y = e^{-2x}$ is a solution of the differential equation $y' + 2y = 0$. It can be shown that every solution of this differential equation is of the form

$$y = Ce^{-2x} \qquad \text{General solution of } y' + 2y = 0$$

where C is any real number. This solution is called the **general solution.** Some differential equations have **singular solutions** that cannot be written as special cases of the general solution. Such solutions, however, are not considered in this text. The **order** of a differential equation is determined by the highest-order derivative in the equation. For instance, $y' = 4y$ is a first-order differential equation. First-order linear differential equations are discussed in Section 6.4.

In Section 4.1, Example 9, you saw that the second-order differential equation $s''(t) = -32$ has the general solution

$$s(t) = -16t^2 + C_1t + C_2 \qquad \text{General solution of } s''(t) = -32$$

which contains two arbitrary constants. It can be shown that a differential equation of order n has a general solution with n arbitrary constants.

EXAMPLE 1 Verifying Solutions

Determine whether the function is a solution of the differential equation $y'' - y = 0$.

a. $y = \sin x$ **b.** $y = 4e^{-x}$ **c.** $y = Ce^x$

Solution

a. Because $y = \sin x$, $y' = \cos x$, and $y'' = -\sin x$, it follows that

$$y'' - y = -\sin x - \sin x = -2 \sin x \neq 0.$$

So, $y = \sin x$ is *not* a solution.

b. Because $y = 4e^{-x}$, $y' = -4e^{-x}$, and $y'' = 4e^{-x}$, it follows that

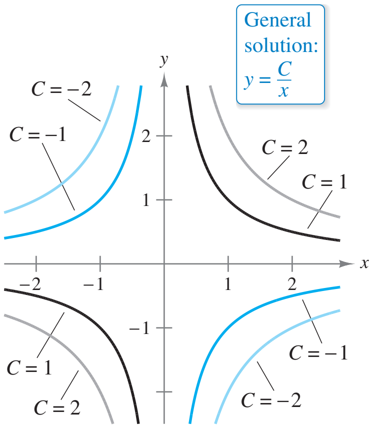
$$y'' - y = 4e^{-x} - 4e^{-x} = 0.$$

So, $y = 4e^{-x}$ is a solution.

c. Because $y = Ce^x$, $y' = Ce^x$, and $y'' = Ce^x$, it follows that

$$y'' - y = Ce^x - Ce^x = 0.$$

So, $y = Ce^x$ is a solution for any value of C.



Solution curves for $xy' + y = 0$
Figure 6.1

Geometrically, the general solution of a first-order differential equation represents a family of curves known as **solution curves,** one for each value assigned to the arbitrary constant. For instance, you can verify that every function of the form

$$y = \frac{C}{x} \qquad \text{General solution of } xy' + y = 0$$

is a solution of the differential equation

$$xy' + y = 0.$$

Figure 6.1 shows four of the solution curves corresponding to different values of C.

As discussed in Section 4.1, **particular solutions** of a differential equation are obtained from **initial conditions** that give the values of the dependent variable or one of its derivatives for particular values of the independent variable. The term "initial condition" stems from the fact that, often in problems involving time, the value of the dependent variable or one of its derivatives is known at the *initial* time $t = 0$. For instance, the second-order differential equation

$$s''(t) = -32$$

having the general solution

$$s(t) = -16t^2 + C_1t + C_2 \qquad \text{General solution of } s''(t) = -32$$

might have the following initial conditions.

$$s(0) = 80, \quad s'(0) = 64 \qquad \text{Initial conditions}$$

In this case, the initial conditions yield the particular solution

$$s(t) = -16t^2 + 64t + 80. \qquad \text{Particular solution}$$

EXAMPLE 2 Finding a Particular Solution

See LarsonCalculus.com for an interactive version of this type of example.

For the differential equation

$$xy' - 3y = 0$$

verify that $y = Cx^3$ is a solution. Then find the particular solution determined by the initial condition $y = 2$ when $x = -3$.

Solution You know that $y = Cx^3$ is a solution because $y' = 3Cx^2$ and

$$xy' - 3y = x(3Cx^2) - 3(Cx^3) = 0.$$

Furthermore, the initial condition $y = 2$ when $x = -3$ yields

$$y = Cx^3 \qquad \text{General solution}$$
$$2 = C(-3)^3 \qquad \text{Substitute initial condition.}$$
$$-\frac{2}{27} = C \qquad \text{Solve for } C.$$

and you can conclude that the particular solution is

$$y = -\frac{2x^3}{27}. \qquad \text{Particular solution}$$

Try checking this solution by substituting for y and y' in the original differential equation. ■

Note that to determine a particular solution, the number of initial conditions must match the number of constants in the general solution.

Slope Fields

Solving a differential equation analytically can be difficult or even impossible. However, there is a graphical approach you can use to learn a lot about the solution of a differential equation. Consider a differential equation of the form

$$y' = F(x, y) \qquad \text{Differential equation}$$

where $F(x, y)$ is some expression in x and y. At each point (x, y) in the xy-plane where F is defined, the differential equation determines the slope $y' = F(x, y)$ of the solution at that point. If you draw short line segments with slope $F(x, y)$ at selected points (x, y) in the domain of F, then these line segments form a **slope field,** or a *direction field*, for the differential equation $y' = F(x, y)$. Each line segment has the same slope as the solution curve through that point. A slope field shows the general shape of all the solutions and can be helpful in getting a visual perspective of the directions of the solutions of a differential equation.

EXAMPLE 3 Sketching a Slope Field

Sketch a slope field for the differential equation $y' = x - y$ for the points $(-1, 1)$, $(0, 1)$, and $(1, 1)$.

Solution The slope of the solution curve at any point (x, y) is

$$F(x, y) = x - y. \qquad \text{Slope at } (x, y).$$

So, the slope at each point can be found as shown.

Slope at $(-1, 1)$: $y' = -1 - 1 = -2$

Slope at $(0, 1)$: $y' = 0 - 1 = -1$

Slope at $(1, 1)$: $y' = 1 - 1 = 0$

Draw short line segments at the three points with their respective slopes, as shown in Figure 6.2.

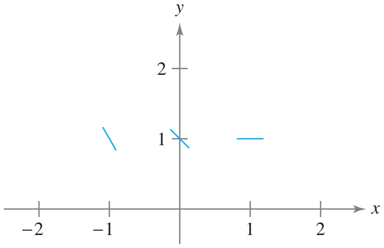
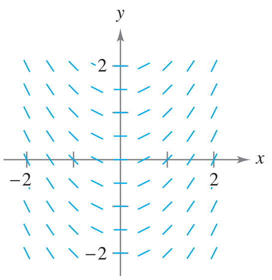


Figure 6.2

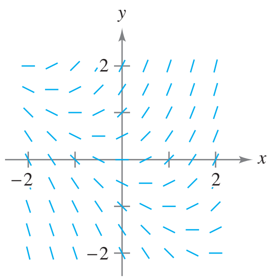
EXAMPLE 4 Identifying Slope Fields for Differential Equations

Match each slope field with its differential equation.

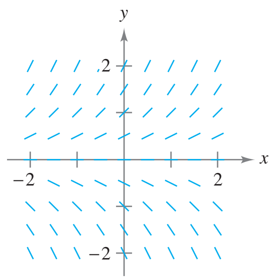
a.



b.



c.



i. $y' = x + y$ **ii.** $y' = x$ **iii.** $y' = y$

Solution

a. You can see that the slope at any point along the y-axis is 0. The only equation that satisfies this condition is $y' = x$. So, the graph matches equation (ii).

b. You can see that the slope at the point $(1, -1)$ is 0. The only equation that satisfies this condition is $y' = x + y$. So, the graph matches equation (i).

c. You can see that the slope at any point along the x-axis is 0. The only equation that satisfies this condition is $y' = y$. So, the graph matches equation (iii).

A solution curve of a differential equation $y' = F(x, y)$ is simply a curve in the xy-plane whose tangent line at each point (x, y) has slope equal to $F(x, y)$. This is illustrated in Example 5.

EXAMPLE 5 Sketching a Solution Using a Slope Field

Sketch a slope field for the differential equation

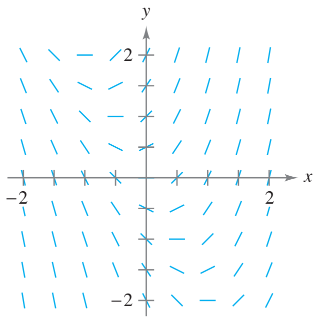
$$y' = 2x + y.$$

Use the slope field to sketch the solution that passes through the point $(1, 1)$.

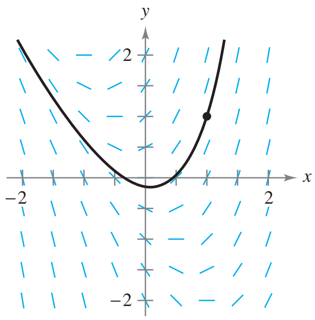
Solution Make a table showing the slopes at several points. The table shown is a small sample. The slopes at many other points should be calculated to get a representative slope field.

x	-2	-2	-1	-1	0	0	1	1	2	2
y	-1	1	-1	1	-1	1	-1	1	-1	1
$y' = 2x + y$	-5	-3	-3	-1	-1	1	1	3	3	5

Next, draw line segments at the points with their respective slopes, as shown in Figure 6.3.



Slope field for $y' = 2x + y$
Figure 6.3

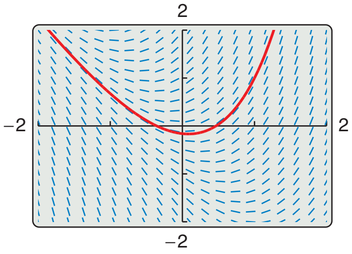


Particular solution for $y' = 2x + y$ passing through $(1, 1)$
Figure 6.4

After the slope field is drawn, start at the initial point $(1, 1)$ and move to the right in the direction of the line segment. Continue to draw the solution curve so that it moves parallel to the nearby line segments. Do the same to the left of $(1, 1)$. The resulting solution is shown in Figure 6.4.

In Example 5, note that the slope field shows that y' increases to infinity as x increases.

TECHNOLOGY Drawing a slope field by hand is tedious. In practice, slope fields are usually drawn using a graphing utility. If you have access to a graphing utility that can graph slope fields, try graphing the slope field for the differential equation in Example 5. One example of a slope field drawn by a graphing utility is shown at the right.



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Euler's Method

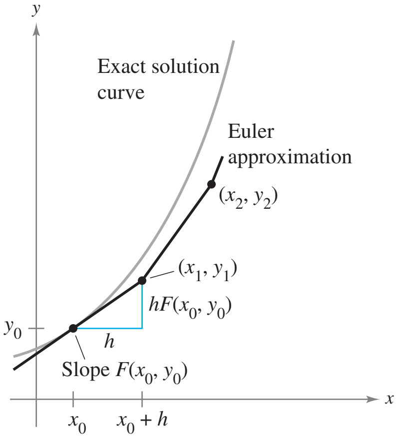


Figure 6.5

Euler's Method is a numerical approach to approximating the particular solution of the differential equation

$$y' = F(x, y)$$

that passes through the point (x_0, y_0). From the given information, you know that the graph of the solution passes through the point (x_0, y_0) and has a slope of $F(x_0, y_0)$ at this point. This gives you a "starting point" for approximating the solution.

From this starting point, you can proceed in the direction indicated by the slope. Using a small step h, move along the tangent line until you arrive at the point (x_1, y_1), where

$$x_1 = x_0 + h \quad \text{and} \quad y_1 = y_0 + hF(x_0, y_0)$$

as shown in Figure 6.5. Then, using (x_1, y_1) as a new starting point, you can repeat the process to obtain a second point (x_2, y_2). The values of x_i and y_i are shown below.

$$\begin{aligned} x_1 &= x_0 + h & y_1 &= y_0 + hF(x_0, y_0) \\ x_2 &= x_1 + h & y_2 &= y_1 + hF(x_1, y_1) \\ &\vdots & &\vdots \\ x_n &= x_{n-1} + h & y_n &= y_{n-1} + hF(x_{n-1}, y_{n-1}) \end{aligned}$$

When using this method, note that you can obtain better approximations of the exact solution by choosing smaller and smaller step sizes.

EXAMPLE 6 Approximating a Solution Using Euler's Method

Use Euler's Method to approximate the particular solution of the differential equation

$$y' = x - y$$

passing through the point $(0, 1)$. Use a step of $h = 0.1$.

Solution Using $h = 0.1$, $x_0 = 0$, $y_0 = 1$, and $F(x, y) = x - y$, you have

$$x_0 = 0, \quad x_1 = 0.1, \quad x_2 = 0.2, \quad x_3 = 0.3,$$

and the first three approximations are

$$\begin{aligned} y_1 &= y_0 + hF(x_0, y_0) = 1 + (0.1)(0 - 1) = 0.9 \\ y_2 &= y_1 + hF(x_1, y_1) = 0.9 + (0.1)(0.1 - 0.9) = 0.82 \\ y_3 &= y_2 + hF(x_2, y_2) = 0.82 + (0.1)(0.2 - 0.82) = 0.758. \end{aligned}$$

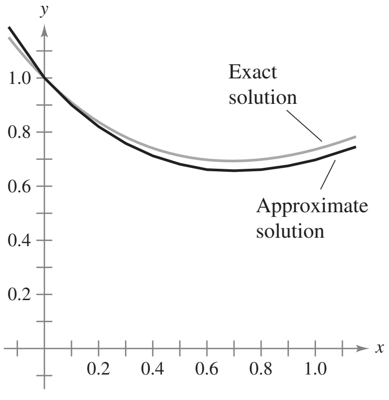


Figure 6.6

The first ten approximations are shown in the table. You can plot these values to see a graph of the approximate solution, as shown in Figure 6.6.

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	1	0.900	0.820	0.758	0.712	0.681	0.663	0.657	0.661	0.675	0.697

For the differential equation in Example 6, you can verify the exact solution to be the equation

$$y = x - 1 + 2e^{-x}.$$

Figure 6.6 compares this exact solution with the approximate solution obtained in Example 6.

6.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Verifying a Solution In Exercises 1–8, verify the solution of the differential equation.

	Solution	Differential Equation
1.	$y = Ce^{4x}$	$y' = 4y$
2.	$y = e^{-2x}$	$3y' + 5y = -e^{-2x}$
3.	$x^2 + y^2 = Cy$	$y' = \dfrac{2xy}{x^2 - y^2}$
4.	$y^2 - 2 \ln y = x^2$	$\dfrac{dy}{dx} = \dfrac{xy}{y^2 - 1}$
5.	$y = C_1 \sin x - C_2 \cos x$	$y'' + y = 0$
6.	$y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$	$y'' + 2y' + 2y = 0$
7.	$y = -\cos x \ln\lvert\sec x + \tan x\rvert$	$y'' + y = \tan x$
8.	$y = \frac{2}{5}(e^{-4x} + e^x)$	$y'' + 4y' = 2e^x$

Verifying a Particular Solution In Exercises 9–12, verify the particular solution of the differential equation.

	Solution	Differential Equation and Initial Condition
9.	$y = \sin x \cos x - \cos^2 x$	$2y + y' = 2\sin(2x) - 1$ $y\left(\dfrac{\pi}{4}\right) = 0$
10.	$y = 6x - 4 \sin x + 1$	$y' = 6 - 4\cos x$ $y(0) = 1$
11.	$y = 4e^{-6x^2}$	$y' = -12xy$ $y(0) = 4$
12.	$y = e^{-\cos x}$	$y' = y \sin x$ $y\left(\dfrac{\pi}{2}\right) = 1$

Determining a Solution In Exercises 13–20, determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

13. $y = 3 \cos x$

14. $y = 2 \sin x$

15. $y = 3 \cos 2x$

16. $y = 3 \sin 2x$

17. $y = e^{-2x}$

18. $y = 5 \ln x$

19. $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \sin 2x + C_4 \cos 2x$

20. $y = 3e^{2x} - 4 \sin 2x$

Determining a Solution In Exercises 21–28, determine whether the function is a solution of the differential equation $xy' - 2y = x^3 e^x$.

21. $y = x^2$

22. $y = x^3$

23. $y = x^2 e^x$

24. $y = x^2(2 + e^x)$

25. $y = \sin x$

26. $y = \cos x$

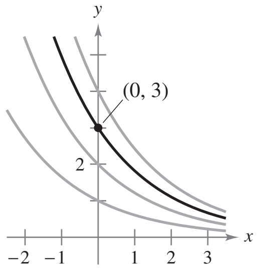
27. $y = \ln x$

28. $y = x^2 e^x - 5x^2$

Finding a Particular Solution In Exercises 29–32, some of the curves corresponding to different values of C in the general solution of the differential equation are shown in the graph. Find the particular solution that passes through the point shown on the graph.

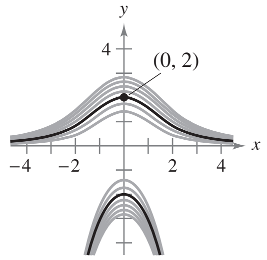
29. $y^2 = Ce^{-x/2}$

$2y' + y = 0$



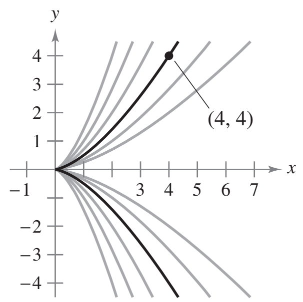
30. $y(x^2 + y) = C$

$2xy + (x^2 + 2y)y' = 0$



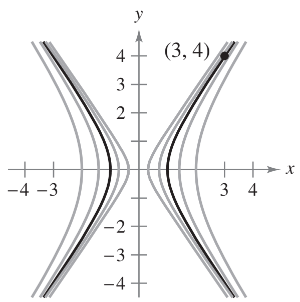
31. $y^2 = Cx^3$

$2xy' - 3y = 0$



32. $2x^2 - y^2 = C$

$yy' - 2x = 0$



Graphs of Particular Solutions In Exercises 33 and 34, the general solution of the differential equation is given. Use a graphing utility to graph the particular solutions for the given values of C.

33. $4yy' - x = 0$

$4y^2 - x^2 = C$

$C = 0, C = \pm 1, C = \pm 4$

34. $yy' + x = 0$

$x^2 + y^2 = C$

$C = 0, C = 1, C = 4$

Finding a Particular Solution In Exercises 35–40, verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition(s).

35. $y = Ce^{-2x}$

$y' + 2y = 0$

$y = 3$ when $x = 0$

36. $3x^2 + 2y^2 = C$

$3x + 2yy' = 0$

$y = 3$ when $x = 1$

37. $y = C_1 \sin 3x + C_2 \cos 3x$

$y'' + 9y = 0$

$y = 2$ when $x = \dfrac{\pi}{6}$

$y' = 1$ when $x = \dfrac{\pi}{6}$

38. $y = C_1 + C_2 \ln x$

$xy'' + y' = 0$

$y = 0$ when $x = 2$

$y' = \dfrac{1}{2}$ when $x = 2$

39. $y = C_1x + C_2x^3$

$x^2y'' - 3xy' + 3y = 0$

$y = 0$ when $x = 2$

$y' = 4$ when $x = 2$

40. $y = e^{2x/3}(C_1 + C_2x)$

$9y'' - 12y' + 4y = 0$

$y = 4$ when $x = 0$

$y = 0$ when $x = 3$

Finding a General Solution **In Exercises 41–52, use integration to find a general solution of the differential equation.**

41. $\dfrac{dy}{dx} = 6x^2$ **42.** $\dfrac{dy}{dx} = 10x^4 - 2x^3$

43. $\dfrac{dy}{dx} = \dfrac{x}{1 + x^2}$ **44.** $\dfrac{dy}{dx} = \dfrac{e^x}{4 + e^x}$

45. $\dfrac{dy}{dx} = \dfrac{x - 2}{x}$ **46.** $\dfrac{dy}{dx} = x \cos x^2$

47. $\dfrac{dy}{dx} = \sin 2x$ **48.** $\dfrac{dy}{dx} = \tan^2 x$

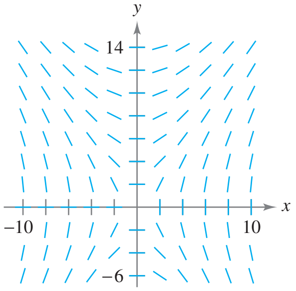
49. $\dfrac{dy}{dx} = x\sqrt{x - 6}$ **50.** $\dfrac{dy}{dx} = 2x\sqrt{4x^2 + 1}$

51. $\dfrac{dy}{dx} = xe^{x^2}$ **52.** $\dfrac{dy}{dx} = 5e^{-x/2}$

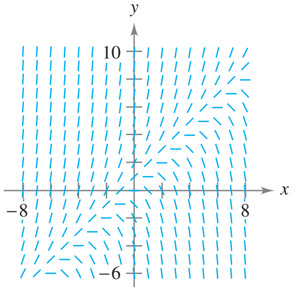
Slope Field **In Exercises 53–56, a differential equation and its slope field are given. Complete the table by determining the slopes (if possible) in the slope field at the given points.**

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx						

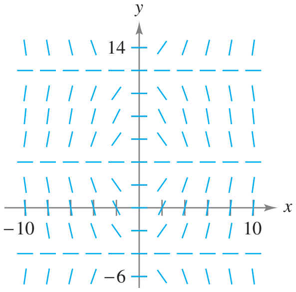
53. $\dfrac{dy}{dx} = \dfrac{2x}{y}$



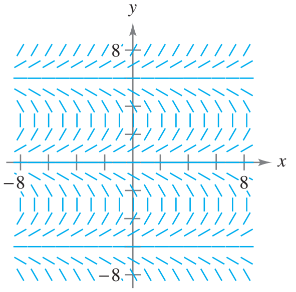
54. $\dfrac{dy}{dx} = y - x$



55. $\dfrac{dy}{dx} = x \cos \dfrac{\pi y}{8}$

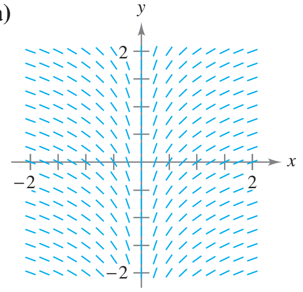


56. $\dfrac{dy}{dx} = \tan\left(\dfrac{\pi y}{6}\right)$

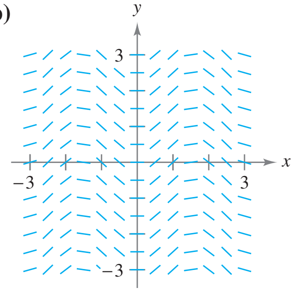


Matching **In Exercises 57–60, match the differential equation with its slope field. [The slope fields are labeled (a), (b), (c), and (d).]**

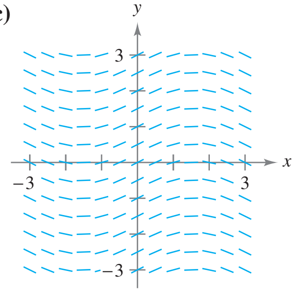
(a)



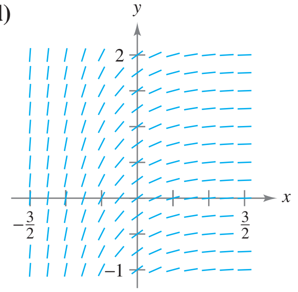
(b)



(c)



(d)



57. $\dfrac{dy}{dx} = \sin(2x)$ **58.** $\dfrac{dy}{dx} = \dfrac{1}{2}\cos x$

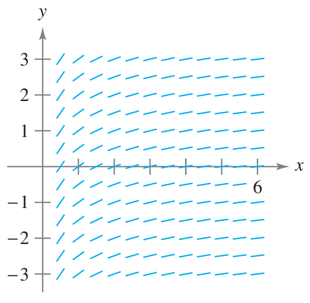
59. $\dfrac{dy}{dx} = e^{-2x}$ **60.** $\dfrac{dy}{dx} = \dfrac{1}{x}$

Slope Field **In Exercises 61–64, (a) sketch the slope field for the differential equation, (b) use the slope field to sketch the solution that passes through the given point, and (c) discuss the graph of the solution as $x \to \infty$ and $x \to -\infty$. Use a graphing utility to verify your results. To print a blank graph, go to *MathGraphs.com*.**

61. $y' = 3 - x$, $(4, 2)$ **62.** $y' = \frac{1}{3}x^2 - \frac{1}{2}x$, $(1, 1)$

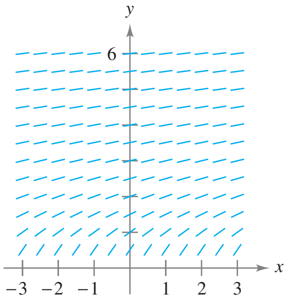
63. $y' = y - 4x$, $(2, 2)$ **64.** $y' = y + xy$, $(0, -4)$

65. Slope Field Use the slope field for the differential equation $y' = 1/x$, where $x > 0$, to sketch the graph of the solution that satisfies each given initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/x$ as $x \to \infty$. To print an enlarged copy of the graph, go to *MathGraphs.com.*



(a) $(1, 0)$ (b) $(2, -1)$

66. **Slope Field** Use the slope field for the differential equation $y' = 1/y$, where $y > 0$, to sketch the graph of the solution that satisfies each given initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/y$ as $x \to \infty$. To print an enlarged copy of the graph, go to *MathGraphs.com*.



(a) $(0, 1)$ (b) $(1, 1)$

Slope Field **In Exercises 67–72, use a computer algebra system to (a) graph the slope field for the differential equation and (b) graph the solution satisfying the specified initial condition.**

67. $\dfrac{dy}{dx} = 0.25y, \quad y(0) = 4$

68. $\dfrac{dy}{dx} = 4 - y, \quad y(0) = 6$

69. $\dfrac{dy}{dx} = 0.02y(10 - y), \quad y(0) = 2$

70. $\dfrac{dy}{dx} = 0.2x(2 - y), \quad y(0) = 9$

71. $\dfrac{dy}{dx} = 0.4y(3 - x), \quad y(0) = 1$

72. $\dfrac{dy}{dx} = \dfrac{1}{2}e^{-x/8} \sin \dfrac{\pi y}{4}, \quad y(0) = 2$

Euler's Method **In Exercises 73–78, use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.**

73. $y' = x + y, \quad y(0) = 2, \quad n = 10, \quad h = 0.1$

74. $y' = x + y, \quad y(0) = 2, \quad n = 20, \quad h = 0.05$

75. $y' = 3x - 2y, \quad y(0) = 3, \quad n = 10, \quad h = 0.05$

76. $y' = 0.5x(3 - y), \quad y(0) = 1, \quad n = 5, \quad h = 0.4$

77. $y' = e^{xy}, \quad y(0) = 1, \quad n = 10, \quad h = 0.1$

78. $y' = \cos x + \sin y, \quad y(0) = 5, \quad n = 10, \quad h = 0.1$

Euler's Method **In Exercises 79–81, complete the table using the exact solution of the differential equation and two approximations obtained using Euler's Method to approximate the particular solution of the differential equation. Use $h = 0.2$ and $h = 0.1$, and compute each approximation to four decimal places.**

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)						
$y(x)$ $(h = 0.2)$						
$y(x)$ $(h = 0.1)$						

Table for 79–81

	Differential Equation	Initial Condition	Exact Solution
79.	$\dfrac{dy}{dx} = y$	$(0, 3)$	$y = 3e^x$
80.	$\dfrac{dy}{dx} = \dfrac{2x}{y}$	$(0, 2)$	$y = \sqrt{2x^2 + 4}$
81.	$\dfrac{dy}{dx} = y + \cos(x)$	$(0, 0)$	$y = \frac{1}{2}(\sin x - \cos x + e^x)$

82. **Euler's Method** Compare the values of the approximations in Exercises 79–81 with the values given by the exact solution. How does the error change as h increases?

83. **Temperature** At time $t = 0$ minutes, the temperature of an object is 140°F. The temperature of the object is changing at the rate given by the differential equation

$$\frac{dy}{dt} = -\frac{1}{2}(y - 72).$$

(a) Use a graphing utility and Euler's Method to approximate the particular solutions of this differential equation at $t = 1$, 2, and 3. Use a step size of $h = 0.1$. (A graphing utility program for Euler's Method is available at the website *college.hmco.com*.)

(b) Compare your results with the exact solution

$$y = 72 + 68e^{-t/2}.$$

(c) Repeat parts (a) and (b) using a step size of $h = 0.05$. Compare the results.

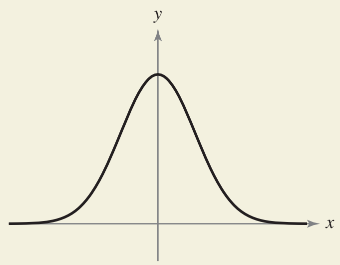
84. **HOW DO YOU SEE IT?** The graph shows a solution of one of the following differential equations. Determine the correct equation. Explain your reasoning.

(a) $y' = xy$

(b) $y' = \dfrac{4x}{y}$

(c) $y' = -4xy$

(d) $y' = 4 - xy$



WRITING ABOUT CONCEPTS

85. General and Particular Solutions In your own words, describe the difference between a general solution of a differential equation and a particular solution.

86. Slope Field Explain how to interpret a slope field.

87. Euler's Method Describe how to use Euler's Method to approximate a particular solution of a differential equation.

88. Finding Values It is known that $y = Ce^{kx}$ is a solution of the differential equation $y' = 0.07y$. Is it possible to determine C or k from the information given? If so, find its value.

True or False? **In Exercises 89–92, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

89. If $y = f(x)$ is a solution of a first-order differential equation, then $y = f(x) + C$ is also a solution.

90. The general solution of a differential equation is $y = -4.9x^2 + C_1x + C_2$. To find a particular solution, you must be given two initial conditions.

91. Slope fields represent the general solutions of differential equations.

92. A slope field shows that the slope at the point $(1, 1)$ is 6. This slope field represents the family of solutions for the differential equation $y' = 4x + 2y$.

93. Errors and Euler's Method The exact solution of the differential equation

$$\frac{dy}{dx} = -2y$$

where $y(0) = 4$, is $y = 4e^{-2x}$.

(a) Use a graphing utility to complete the table, where y is the exact value of the solution, y_1 is the approximate solution using Euler's Method with $h = 0.1$, y_2 is the approximate solution using Euler's Method with $h = 0.2$, e_1 is the absolute error $|y - y_1|$, e_2 is the absolute error $|y - y_2|$, and r is the ratio e_1/e_2.

x	0	0.2	0.4	0.6	0.8	1
y						
y_1						
y_2						
e_1						
e_2						
r						

(b) What can you conclude about the ratio r as h changes?

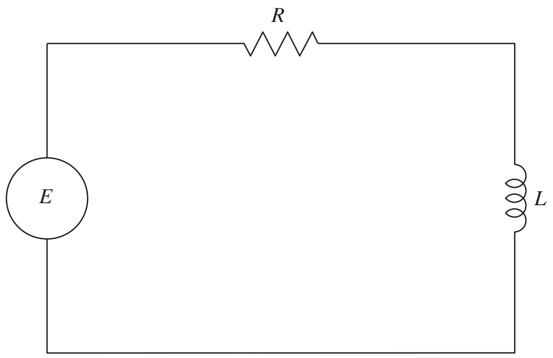
(c) Predict the absolute error when $h = 0.05$.

94. Errors and Euler's Method Repeat Exercise 93 for which the exact solution of the differential equation

$$\frac{dy}{dx} = x - y$$

where $y(0) = 1$, is $y = x - 1 + 2e^{-x}$.

95. Electric Circuit The diagram shows a simple electric circuit consisting of a power source, a resistor, and an inductor.



A model of the current I, in amperes (A), at time t is given by the first-order differential equation

$$L\frac{dI}{dt} + RI = E(t)$$

where $E(t)$ is the voltage (V) produced by the power source, R is the resistance, in ohms (Ω), and L is the inductance, in henrys (H). Suppose the electric circuit consists of a 24-V power source, a 12-Ω resistor, and a 4-H inductor.

(a) Sketch a slope field for the differential equation.

(b) What is the limiting value of the current? Explain.

96. Think About It It is known that $y = e^{kt}$ is a solution of the differential equation $y'' - 16y = 0$. Find the values of k.

97. Think About It It is known that $y = A \sin \omega t$ is a solution of the differential equation $y'' + 16y = 0$. Find the values of ω.

PUTNAM EXAM CHALLENGE

98. Let f be a twice-differentiable real-valued function satisfying

$$f(x) + f''(x) = -xg(x)f'(x)$$

where $g(x) \geq 0$ for all real x. Prove that $|f(x)|$ is bounded.

99. Prove that if the family of integral curves of the differential equation

$$\frac{dy}{dx} + p(x)y = q(x), \quad p(x) \cdot q(x) \neq 0$$

is cut by the line $x = k$, the tangents at the points of intersection are concurrent.

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